Revisiting Characteristic Impedance and Its Definition of Microstrip Line with a Self-Calibrated 3-D MoM Scheme

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***Abstract*—Characteristic impedance and its definition are revisited and discussed for microstrip line with a self-calibrated three-dimenional (3-D) method of moments (MoM). This 3-D MoM accommodates a scheme called short-open calibration (SOC) so that potential parasitic effects brought by the impressed voltage excitation and other relevant factors can be effectively removed. In this way, the characteristic impedance can be accurately defined through a relationship between equivalent voltage and current on the two sides of a microstrip line having a finite length. Simulated results are compared with the Jansen's two-dimensional (2-D) and Rautio's 3-D definition.**

***Index Terms*—Characteristic impedance, method of moments, microstrip line.**

I. Introduction

THE three-dimensional (3-D) deterministic method of moments (MoM) using the impressed-voltage-source (IVS) [1], [3], [4] or impressed-current-source (ICS) excitation [2] has been attractive for modeling and design of shielded multilayered planar circuits. As usual, a de-embedding procedure [5] is needed to transform the calculated results at the source plane of feed line, far away from the discontinuity, to the reference plane of interest, the input/output of a device under test (DUT), to name an example. To do so, the two-dimensional (2-D) characteristic impedance definition of a uniform microstrip line with infinite length [6] is chosen to describe the 3-D propagation characteristics of relevant feed lines with a finite length located between the source plane and the reference plane. The problem of inconsistency between these 2-D/3-D definitions brings about some of difficult-to-estimate effects on the de-embedded circuit parameter. It is because these potential effects, even though eventually very small, are basically frequency-/dimension-dependent. Therefore, the 3-D impedance definition of a microstrip line is required in the accurate de-embedding of this core part of a planar circuit.

To evaluate this problem, Rautio [7] came up with a novel TEM equivalent impedance definition in the modeling of a uniform line with finite length. This is done by formulating the port discontinuity as a lumped shunt capacitor and removing its effect from the 3-D MoM results. However, several potential errors may still affect the accuracy such as the approximate model of the port discontinuity at the source plane.

By analog to the well-known through-reflect-line (TRL) calibration procedure developed in the microwave measurement [8], the entire structure of a circuit is divided into two parts in our proposed IVS-MoM algorithm [9]: *feed lines* and *discontinuity*, which are equivalent to error box and DUT in the TRL calibration. In our case, the relevant circuit block can be precisely extracted by evaluating the feed line boxes through the application of a so-called short-open calibration (SOC) technique [10], [11]. Similar to [8], the characteristic impedance and effective dielectric constant can be determined through 3-D model.

II. Characteristic Impedance and Its Definitions

Fig. 1 shows the 3-D model for calculating the characteristic impedance of a uniform microstrip line by using the IVS-MoM algorithm [9] with the proposed SOC technique [10], [11]. The key issue is to establish an equivalent network for this uniform line such that the equivalent voltage and current defined at specific planes can be interrelated to each other. Such a relationship usually leads to a chain matrix which gives rise to the characteristic impedance and effective dielectric constant. The interesting features of this definition lie in its measurement compatibility and its ability to account for dispersion effect. Also, this definition can easily be applied to non-TEM lines. In use of the IVS-MoM algorithm with the SOC technique, a pair of IVS are launched at the two ports, namely, #1 and #2, of the microstrip line backed by electric walls. As in the TRL calibration [8], the entire line is partitioned into the feed lines (error boxes) and the uniform line (DUT) as shown in Fig. 1(a). The electric walls can be effectively removed by introducing an image principle and details on our IVS-MoM implementation can be found in [9]–[11].

Fig. 2(a) depicts the related equivalent circuit model of Fig. 1(a). V_1, I_1, V_2, and I_2 stand for the port voltages and currents at the planes denoted #1 and #2 while V_1', I_1', V_2', and I_2' are the equivalent voltages and currents at the reference planes #1$'$ and #2$'$ of the given uniform line section to be derived by the SOC technique. The uniform line should be modeled by complex characteristic impedance and propagation constant for unbounded environment considering potential radiation/leakage effects. In the following, only the real parts will be given to compare with others. The error boxes in Fig. 2(a) account for unwanted parasitic effects essentially

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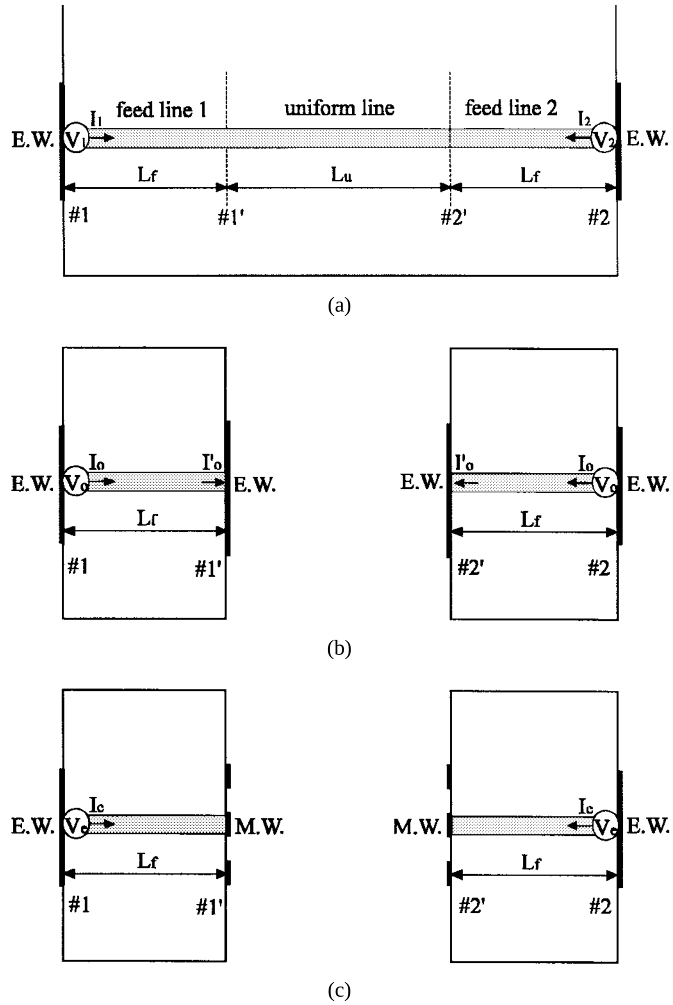


Fig. 1. 3-D model for the derivation of characteristic impedance of the uniform microstrip line with finite length L, based on the self-calibrated IVS-MoM technique. (a) Physical model. (b) Short element. (c) Open element.

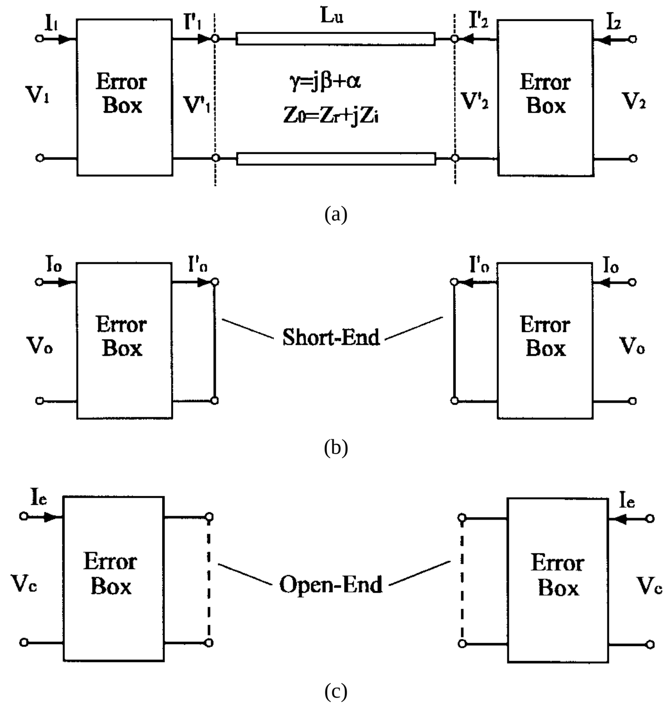


Fig. 2. Equivalent circuit network of the proposed short-element and open-element 3-D model, described in Fig. 1. (a) Physical model. (b) Short element. (c) Open element.

brought by the approximation of the excitation model at the source plane [7] and also the solution consistency problem between the 2-D and 3-D simulations.

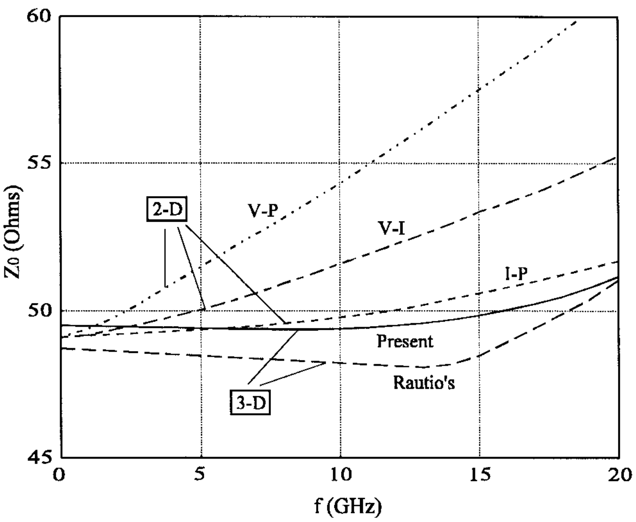
The application of the SOC technique requires the setting up of two standard elements, namely, *short* and *open*, which are separately formulated by the uniform lines attached with the feed lines and loaded with the ideal electric/magnetic walls (E. W. and M. W.) at the reference planes #1$'$ and #2$'$, as described schematically in Fig. 1(b) and (c). Fig. 2(b) and (c) represents the equivalent circuit network of these two standard elements.

With reference to [9]–[11], the relationship between the port voltages and currents can be numerically formulated by solving a source-type matrix equation. Similarly, the currents I_e and I_o flowing at the ports and I_o' flowing at the short-end of Fig. 1(b) can be expressed as a function of even/odd impressed voltages V_e and V_o by solving a pair of matrix equations related to the short and open elements defined in Fig. 1(b) and (c). Therefore, parameters of the error boxes as shown in Fig. 2(b) and (c) can be accurately evaluated and explicitly formulated as a ABCD matrix $[A_{eb}]$ in terms of the normalized even/odd currents with respect to the even/odd impressed voltages [10], [11].

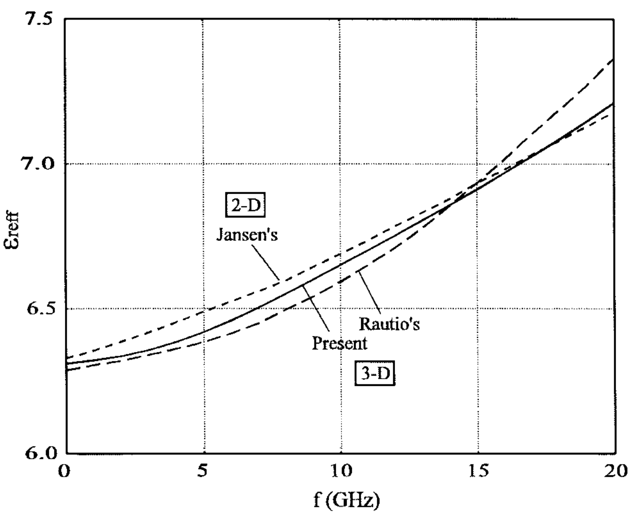
The equivalent ABCD matrix $[A]$ of the uniform line in Fig. 2(a) can be derived by analytically converting the currents/voltages at the ports #1 and #2 to the reference planes #1$'$ and #2$'$ through the calculated matrix of the error boxes $[A_{eb}]$, such that the characteristic impedance Z_0 and the phase constant β of such a uniform line with the length L can be explicitly derived from the calculated ABCD matrix elements.

III. Results and Discussion

Fig. 3 shows simulated results based on the 3-D definition using our proposed self-calibrated MoM technique compared with three 2-D classical definitions [6] and the 3-D model derived by Rautio [5], [7]. In our wideband simulation from $f = 0.1$–20 GHz. the entire conductor strip is discretized by the same mesh scheme along the transverse/longitudinal directions regardless of frequency to achieve the continuity of calculated curves. The length of the feed lines and the uniform line in the model of Fig. 1(a) are numerically characterized by 20 and 40 uniform meshes with the mesh size of 0.635 mm, respectively, while the strip width by five meshes. In Fig. 3(a), five groups of calculated characteristic impedance are plotted for the three 2-D definitions, namely, voltage-power (V-P), voltage-current (V-I), current-power (I-P) [6], and the 3-D definitions based on ABCD matrix proposed by Rautio [5], [7] and the present work. It can be seen that all the three groups of 2-D curves increase monotonously with frequency and they tend to be rapidly different at high frequency. Obviously, different consideration of dispersion effect in these definitions yields such a difference. On the other hand, the 3-D results show quite a flat behavior at relatively low frequency. In [7],



(a)



(b)

Fig. 3. Systematic comparison among simulated results on the characteristic impedance (Z_o) and effective dielectric constant (ε_{reff}), given by the 2-D classical definitions, the 3-D Rautio's technique, and our proposed 3-D technique for a microstrip line ($w = h = 0.635$ mm, $\varepsilon_r = 9.7$). (a) Characteristic impedance (Z_o). (b) Effective dielectric constant (ε_{reff}).

Rautio even attempted to give two groups of measured curves to validate this behavior.

It is commonly accepted that the I-P definition is most suitable for the characteristic impedance of a microstrip line since the longitudinal current flowing on the strip can be physically interpreted. This is also confirmed by the 3-D definitions, whose results are much closer to the I-P curve than the other two 2-D definitions at high frequency. In addition, it is found that our results are in better agreement with the I-P definition with relative difference lower than 2.0% over the entire frequency range of interest as compared to the Rautio's. This is probably, in our opinion, due to the fact that there are certain unsolved issues in the 3-D de-embedding technique reported in [7], for example, the port discontinuity approximately modeled as a lumped shunt capacitor. Strictly speaking, this discontinuity should be handled by a pair of frequency-dependent shunt capacitance and series inductance. Nevertheless, it can be generalized that the 3-D model leads to better impedance definition and should provide a generalized solution for the characteristic impedance.

Fig. 3(b) shows the comparison among the effective dielectric constant for the same microstrip line, calculated by the Jansen's 2-D model, Rautio's 3-D model, and our 3-D model. All the curves are found to increase with frequency and they are in good agreement with each other. This indicates that the effective dielectric constant is not so problematic as the characteristic impedance.

IV. Conclusion

An accurate de-embedding technique under the self-calibrated IVS-MoM scheme with SOC technique is proposed for the modeling of a uniform microstrip line with finite length in terms of its characteristic impedance and effective dielectric constant. The 3-D definition of characteristic impedance as compared to its 2-D counterparts seems to be more adequate in removing ambiguity and may lead to more consistent results for modeling and design of multilayered planar circuits and antennas at low and high frequency. The results indicate also that the commonly used I-P definition for the microstrip line is also an appropriate choice. The effective dielectric constant based the 2-D and 3-D definitions is also examined and found to be consistent for all the definitions.

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